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COMPARISON R AND CURLI METHODS FOR MULTI-CRITERIA DECISION MAKING

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Abstract:
When multi-criteria decision making, decision makers will expend significant effort in selecting a data normalization method and a weighting method. If a mistake is made in those choices, it will result in decisions that do not find the best solution. Furthermore, with qualitative criteria, it is impossible to standardize the data. Similarly, determining the weights of criteria will be difficult if the criteria are in qualitative form. R and CURLI are two multi-criteria decision-making methods that do not require data normalization or the use of additional weighting methods for the criteria. They are therefore well suited for ranking alternatives when the criteria are both quantitative and qualitative. This study compares the two methods through three examples from different fields. The results show that these two methods jointly determine the best solution in all three fields and are also suitable when using other decision-making methods that require data normalization and have high requirements using the method of determining the weights for the criteria.

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1. INTRODUCTION

This Multi-criteria decision making methods are useful tools for decision makers to select the best option out of a set of alternatives. Over time, many multi-criteria decision-making methods have been proposed by scientists. In each of these methods, the implementation method may be partially or completely different. There are methods that require both normalization of the data and methods of determining the weights of criteria. There are methods that do not require data normalization. Besides, there are methods that do not require determining weights for the criteria. In addition, there are methods that do not require both normalization of data and methods of determining weights for the criteria. For naming convenience, methods that require both data normalization and weighting belong to group A. Methods requiring only data normalization belong to group B and methods that do not need both normalizations of data and methods of determining the weights belong to group C. Whether it is necessary to do so for some multi-criteria decision-making methods are presented in Table 1.

In Table 1, only a part of hundreds of multi-criteria decision-making methods are listed. However, it is found that the number of methods belonging to group A is much larger than that of the methods of group B and group C. The methods of group A have been used a lot for multi-criteria decision making in many fields, in different areas. Nevertheless, it is noticed that data normalization can only be done for numbers, i.e. criteria must be in quantitative form. When the indicators are qualitative, data normalization will not be possible. This is the first limitation of the methods of group A. When all criteria are quantitative, methods of group A may be used.

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Table 1. The need for data normalization and additional use of weighting methods when using multi-criteria decision-making methods

Multi-Criteria Decision-Making Method	Symbol	Data normalization requesting	Necessary to define weights for the criteria	Reference
Multiobjective Optimization On the basis of Ratio Analysis	MOORA	*yes*	*yes*	[1]
COmplex PRroportional ASsessment	COPRAS	*yes*	*yes*	[2]
Simple Additive Weighting	SAW	*yes*	*yes*	[3]
Weighted Aggregates Sum Product ASsessment	WASPAS	*yes*	*yes*	[4]
Proximity Indexed Value	PIV	*yes*	*yes*	[5]
Technique for Order Preference by Similarity to Ideal Solution	TOPSIS	*yes*	*yes*	[6]
VIsekriterijumska optimizacijaI KOmpromisno Resenje (in Serbian)	VIKOR	*yes*	*yes*	[7]
Multi-Attributive Border Approximation area Comparison	MABACH	*yes*	*yes*	[8]
Multi Atributive Ideal-Real Comparative Analysis	MAIRCA	*yes*	*yes*	[9]
Measurement of Alternatives and Ranking according to Compromise Solution	MARCOS	*yes*	*yes*	[10]
Evaluation by an Area-based Method of Ranking	EAMR	*yes*	*yes*	[11]
Preference Analysis for Reference Ideal Solution	PARIS	*yes*	*yes*	[12]
Ranking of Alternatives through Functional mapping of criterion sub-intervals into a Single Interval	RAFSI	*yes*	*yes*	[13]
Combined Compromise Solution	COCOSO	*yes*	*yes*	[14]
COmbinative Distance-based ASsessment	CODAS	*yes*	*yes*	[15]
Preference Selection Index	PSI	*yes*	*No*	[16]
Pareto-Edgeworth Grierson	PEG	*yes*	*No*	[17]
It is based on ranking of the attributes and alternatives	R	*No*	*No*	[18]
Collaborative Unbiased Rank List Integration	CURLI	*No*	*No*	[19,20]

However, another problem arises is the selection of the method of data normalization suitable for the decision-making method. That is because the method of data normalization has a great influence on the results of ranking options [21-23]. The inappropriate selection of the data normalization method also causes inversion of the rankings of the alternatives, and the best method is easily ignored [24-26]. In order to choose a method of data normalization, it is necessary to use multiple methods of data normalization at the same time in each specific case, which consumes effort and time for decision makers. This is not appropriate in urgent cases, such as the selection of solutions to combat the current Covid-19 epidemic, where the whole world is suffering the heavy consequences caused by it. This is the second limitation of the methods of group A. On the other hand, when using methods of group A, it is necessary to use an additional weighting method for the criteria. However, the ranking results of the alternatives also depend a lot on the method of determining the weights [27,28].

Having to use multiple weighting methods simultaneously to choose the most suitable method also consumes time and effort for decision makers. This is the third limitation of the methods of group A. For the methods of group B, since data normalization is still required, the limitations for group B methods are the same as for group A methods (only the third restriction is reduced because there is no need to determine the weights of criteria). The three limitations mentioned above will be eliminated as if using the methods of group C. In Table 1, the two methods belonging to group C are the R method and the CURLI method.

The R method is proposed in 2021 [18]. This is a method that does not need to normalize the data and does not need to use a separate method of determining the weights. In this method, the weight of the criteria is determined in a special way, that is, based on the priority of the criteria as well as on the ranking of alternatives for each criterion. More details on this method are presented in part 2 of this paper. When the criteria are both qualitative and quantitative, this method can be used to rank alternatives. This method has been used for supplier selection, industrial robot selection, material selection for a given industrial product, and flexible manufacturing system selection [18]. So far, there have not been any more studies that have applied this method to multi-criteria decision making.

The CURLI method was first proposed in 2016 for the selection of applicants to a medical school [19]. This method performs the ranking of alternatives based on the comparison of the value of the criteria in each alternative. Normalizing the data and determining the weights for the criteria is not necessary when using this method. Therefore, this method is also suitable to be used for multi-criteria decision making when the criteria are in both quantitative and qualitative form. In Section 2 will present the steps to follow this method. However, in the following years, no studies on applying this method were published. More recently, this method has been elucidated by my self. When applying this method to multi-criteria decision making of turning process, I has been found that this method is equivalent to PEG method, and better than PSI method [20].

From the above analysis, it can be seen that when the criteria are qualitative, it is difficult to use methods belonging to group A and group B, which three basic difficulties have been mentioned above. These difficulties will not be encountered if R method and CURLI method are used. However, the number of studies that have used these two methods is extremely rare. The comparison of these two methods in one study is very novel and it will be done in this study. In order to draw general conclusions when comparing these two methods, three examples from three different fields will be made in part 3 of this paper. The number of options, the number of criteria as well as the form of the criteria (qualitative or quantitative) in the cases are also different.

2. R AND CURLI METHODS

R method

In [18], the steps to perform the ranking of alternatives were presented by the guide in the form of paragraphs (excluding formulas). Perhaps this is the reason why this method has not received much attention from scientists. Those passages, if described as formulas, would make it more accessible. In this study, the passages were refined into specific steps in the form of formulas to make them easier to understand. The steps are as follows:

Step 1. Establish the decision matrix as shown in Table 2. Where m is the number of alternatives, n is the number of criteria. The criteria can be both quantitative and qualitative.

Table 2. Decision Matrix

No.	C_1	C_2	C_i	C_n
A_1	x_{11}	x_{12}	x_{1j}	x_{1n}
A_2	x_{21}	x_{22}	x_{2j}	x_{2n}
A_i	x_{i1}	x_{i2}	x_{ij}	x_{im}
A_m	x_{m1}	x_{m2}	x_{mj}	x_{mn}

Step 2. Make a decision to rank the criteria based on its importance.

Step 3. Rank the alternatives for each criterion.

Step 4. Convert the ranks into corresponding weights according to formula (1).

$$w^{(k)} = \frac{1}{1+\frac{1}{2}+\cdots+\frac{1}{r_k}} \tag{1}$$

Where r_k is the value of rank k, k = 1÷max(m, n). Example for 1st rank: $w^{(1)} = 1/1 = 1$; for 2nd rank: $w^{(2)} = 1/(1+1/2) = 0.6667$; for 3rd rank: $w^{(3)} = 1/(1+1/2+1/3) = 0.5455$; ect.

Step 5. Assign weights to ranked ranks according to formula (2).

$$w^{(r_k)} = \frac{w^{(k)}}{\sum_{k=1}^{max(m,n)} w^{(k)}} \tag{2}$$

Where k = 1÷max(m, n).

For example, if there are 3 options (m = 3) and 4 criteria (n = 4). We have:

$$w^{(1)} = 1/1 = 1$$
$$w^{(2)} = 1/(1+1/2) = 0.6667$$
$$w^{(3)} = 1/(1+1/2+1/3) = 0.5455$$
$$w^{(4)} = 1/(1+1/2+1/3+1/4) = 0.48$$

So:

$$\sum_{k=1}^{\max(m,n)} w^{(k)} = 1+0.6667+0.5455+0.48 = 2.6922$$

Then:

$$w^{(r_1)} = 1/2.6922 = 0.3714$$
$$w^{(r_2)} = 0.6667/2.6922 = 0.2476$$
$$w^{(r_3)} = 0.5455/2.6922 = 0.2026$$
$$w^{(r_4)} = 0.48/2.6922 = 0.1783$$

Step 6. Calculate the points for each alternative using the formula (3).

$$S_i = \sum_{j=1}^{n} w_j^{(k)} \cdot w_j^{(r_k)}, i = 1 \div m \qquad (3)$$

Step 7. Rank the alternatives according to the value of S_i. The one with the largest S_i is the best one.

CURLI method

Rank the alternatives using the CURLI method according to the following steps [19,20]:

Step 1. Similar to step 1 of method R.

Step 2. Create a square matrix of level m as shown in Table 3. Scoring each cell of the matrix in the following way: For example, in column 1 where A1 is better than A2, then fill in the number 1 in the corresponding cell from column 1 - row 2; In column 2 where A2 is worse than A1, enter the number -1 in the cell corresponding to column 2 - row 1; in column 2 where A2 is equal to Am, enter 0 in the cell corresponding to column 2 - row m; in the cells of the main diagonal of the matrix (cells 1-1, cells 2-2, cells 3-3...) are left blank. For n criteria it is necessary to construct n scoring matrices in this way.

Table 3. Example scoring matrix for each indicator

No.	P_1	P_2	...	P_m
A_1		-1	...	...
A_2	1		...	...
...	...	...		...
A_m	...	0	...	

Step 3. Add up all the scoring matrices for the criteria scored by the process scoring matrix.

Step 4. Change the positions of rows and columns in the process scoring matrix so that all elements above the main diagonal are less than or equal to 0. Then the option in row 1 is the best option. That means the ranking order of alternatives is the order of rows from top to bottom.

3. EXAMPLE TO COMPARE R AND CURLI METHOD

3.1 Example 1

Data on Robot types were used in this example [29]. There are seven types of robots with five criteria: Load Capacity (LC), Maximum tip Speed (MS), REpeatability (RE), Memory Capacity (MC), and Manipulator Reach (MR). Among these criteria, LC, MS, MC and MR are the criteria as large as possible, and RE is the criterion as small as possible (Table 4). According to experts, the number one importance belongs to MC criteria, followed by the importance of LC, SM, RE criteria, and the importance of MR ranks fifth.

Table 4. Data for example 1 [29]

No.	Robots	Criteria				
		LC (kg)	SM (mm/s)	RE (mm)	MC (step)	MR (mm)
A_1	ASEA-IRB 60/2 Cincinnati	60	0.4	2540	500	990
A_2	Milacrone T3-726 Cybotech V15	6.35	0.15	1016	3000	1041
A_3	Electric Robot Hitachi America	6.8	0.1	1727.2	1500	1676
A_4	Process Robot Unimation PUMA	10	0.2	1000	2000	965
A_5	Unimation PUMA 500/600	2.5	0.1	560	500	915
A_6	United States Robots Maker 110	4.5	0.08	1016	350	508
A_7	Yaskawa Electric Motoman L3C	3	0.1	1778	1000	920

Apply R method

The decision matrix is created by the last 5 columns in Table 4.

The ranking of options for each criterion (LC, SM, RE, MC and MR) is shown in Table 5. In which it should be noted as follows: The value of RE at A2 and A6 are equal, so the two options are equal, these two alternatives are ranked at 3.5 (the average of 3 and 4); Since the values of MC at A1 and A5 are equal, these two alternatives are ranked at 5.5 (the average of 5 and 6).

Converting the ranks to the weighted form according to formula (1) will get the results as shown in Table 6.

Assigning weights to the ratings according to formula (2), the results are shown in Table 7.

According to Table 7, the weights of the criteria LC, SM, RE, MC and MR are 0.1699, 0.1390, 0.1223, 0.2548, and 0.1116, respectively.

Calculating the Si score for each option according to formula (3) will give the results as shown in Table 8. The results of ranking the options according to the value of Si have also been summarized in this table.

Table 5. Rank the alternatives for each criterion

No.	*LC*	*SM*	*RE*	*MC*	*MR*
A_1	1	1	7	5.5	3
A_2	4	3	3.5	1	2
A_3	3	5	5	3	1
A_4	2	2	2	2	4
A_5	7	5	1	5.5	6
A_6	5	7	3.5	7	7
A_7	6	5	6	4	5

Table 6. Weights are converted from ranks

	k						
Rank k	1	2	3	4	5	6	7
$w^{(k)}$	1.0000	0.6667	0.5455	0.4800	0.4380	0.4082	0.3857

Table 7. Weights assigned to ranks

	k						
Rank k	1	2	3	4	5	6	7
$w^{(r_k)}$	0.2548	0.1699	0.1390	0.1223	0.1116	0.1040	0.0983

Table 8. S_i score for each option and rank

No.	$w_j^{(k)} \cdot w_j^{(r_k)}$					*Si*	*Rank*
	LC	*SM*	*RE*	*MC*	*MR*		
A_1	0.0433	0.0354	0.0120	0.0275	0.0155	0.1337	2
A_2	0.0208	0.0193	0.0160	0.0649	0.0190	0.1400	1
A_3	0.0236	0.0157	0.0137	0.0354	0.0284	0.1168	4
A_4	0.0289	0.0236	0.0208	0.0433	0.0137	0.1302	3
A_5	0.0167	0.0157	0.0312	0.0275	0.0116	0.1026	5
A_6	0.0190	0.0137	0.0160	0.0250	0.0110	0.0846	7
A_7	0.0177	0.0157	0.0127	0.0312	0.0125	0.0897	6

Apply CURLI method

Building a scoring matrix for each criterion LC, SM, RE, MC and MR obtained the corresponding results in Tables 9, 10, 11, 12 and 13. Add the scoring matrices for each criterion in Tables 9, 10, 11, 12 and 13 to get the scoring matrix for the process as shown in Table 14.

Reposition the rows and columns in Table 14 so that all elements above the main diagonal are less than or equal to 0 and get the result as shown in Table 15.

Table 9. Scoring matrix for criteria LC

No.	P_1	P_2	P_3	P_4	P_5	P_6	P_7
A_1		-1	-1	-1	-1	-1	-1
A_2	1		1	1	-1	-1	-1
A_3	1	-1		1	-1	-1	-1
A_4	1	-1	-1		-1	-1	-1
A_5	1	1	1	1		1	1
A_6	1	1	1	1	-1		-1
A_7	1	1	1	1	-1	1	

Table 10. Scoring matrix for criteria SM

No.	P_1	P_2	P_3	P_4	P_5	P_6	P_7
A_1		-1	-1	-1	-1	-1	-1
A_2	1		-1	1	-1	-1	-1
A_3	1	1		1	0	-1	0
A_4	1	-1	-1		-1	-1	-1
A_5	1	1	0	1		-1	0
A_6	1	1	1	1	1		1
A_7	1	1	0	1	0	-1	

Table 11. Scoring matrix for criteria RE

No.	P_1	P_2	P_3	P_4	P_5	P_6	P_7
A_1		1	1	1	1	1	1
A_2	-1		-1	1	1	0	-1
A_3	-1	1		1	1	1	-1
A_4	-1	-1	-1		1	-1	-1
A_5	-1	-1	-1	-1		-1	-1
A_6	-1	0	-1	1	1		-1
A_7	-1	1	1	1	1	1	

Table 12. Scoring matrix for criteria MC

No.	P_1	P_2	P_3	P_4	P_5	P_6	P_7
A_1		1	1	1	0	-1	1
A_2	-1		-1	-1	-1	-1	-1
A_3	-1	1		1	-1	-1	-1
A_4	-1	1	-1		-1	-1	-1
A_5	0	1	1	1		-1	1
A_6	1	1	1	1	1		1
A_7	-1	1	1	1	-1	-1	

Table 13. Scoring matrix for criteria RM

No.	P_1	P_2	P_3	P_4	P_5	P_6	P_7
A_1		1	1	-1	-1	-1	-1
A_2	-1		1	-1	-1	-1	-1
A_3	-1	-1		-1	-1	-1	-1
A_4	1	1	1		-1	-1	-1
A_5	1	1	1	1		-1	1
A_6	1	1	1	1	1		1
A_7	1	1	1	1	-1	-1	

Table 14. Process Scoring Matrix

No.	P_1	P_2	P_3	P_4	P_5	P_6	P_7
A_1		1	1	-1	-2	-3	-1
A_2	-1		-1	1	-3	-4	-5
A_3	-1	1		3	-2	-3	-4
A_4	1	-1	-3		-3	-5	-5
A_5	2	3	2	3		-3	2
A_6	3	4	3	5	3		1
A_7	1	5	4	5	-2	-1	

Table 15. Scoring matrix of the process after changing the positions of rows and columns

No.	P_2	P_1	P_4	P_3	P_5	P_7	P_6
A_2		-1	-1	-1	-3	-5	-4
A_1	1		-1	1	-2	-1	-3
A_4	-1	1		-3	-3	-5	-5
A_3	1	1	3		-2	-4	-3
A_5	3	2	3	2		-1	-3
A_7	5	1	5	4	2		-1
A_6	4	3	5	3	3	1	

From Table 15, the ranking results of the options are as follows: A2 > A1 > A4 > A3 > A5 > A7 > A6.

Thus, R and CURLI methods have been used to rank the alternatives for Example 1. The results are summarized in Table 16. The results of ranking options by CODAS methods [29] have also summarized in this table.

According to Table 16, the ranking results of the alternatives are not the same when implemented by different methods. This has also been confirmed in many studies [21-26]. However, both methods used in this study (including R and CURLI) indicate that A2 is the best option. Especially, A2 has also been confirmed as the best option when using CODAS method [29]. Another interesting thing that has also been confirmed is that the results of ranking options by the two methods R and CURLI are completely identical. From that, it can be confirmed that in this example, the two methods R and CURLI are equally effective.

Table 16. Results of ranking options according to different methods

No.	R	*CURLI*	*CODAS [29]*
A_1	2	2	3
A_2	1	1	1
A_3	4	4	2
A_4	3	3	5
A_5	5	5	7
A_6	7	7	6
A_7	6	6	4

3.2 Example 2

Experimental data for a turning process were used in this example (Table 17) [30]. Nine experiments of a turning process were performed. Four criteria to evaluate the turning process include three components of cutting force Fx, Fy, Fz, and MRR. The unit of the three shear force components is N, the unit of MRR is mm^3/s. Fx, Fy and Fz belong to the group of criteria as small as possible, whereas MRR belongs to the group of criteria as large as possible. According to some experts, the importance of four criteria is determined in order of priority as follows: number one belongs to MRR, priority number two belongs to Fx, priority number three belongs to Fy, and Fz receives fourth priority. The task of the multi-criteria decision problem is to determine the Ai option that simultaneously guarantees the smallest Fx, Fy, Fz and the largest MRR.

Table 17. Data for example 2 [30]

No.	F_x	F_y	F_z	*MRR*
A_1	59.844	187.437	44.165	11.561
A_2	87.943	199.762	99.125	49.062
A_3	78.913	127.456	69.874	109.108
A_4	54.816	172.714	60.19	28.588
A_5	63.117	180.361	68.869	99.039
A_6	68.79	113.951	70.694	61.669
A_7	46.654	116.88	92.222	57.177
A_8	44.989	162.337	63.25	55.462
A_9	54.846	167.837	74.165	151.09

The ranking of the options in Table 17 by two methods R and CURLI is similar to example 1 and the results are presented in Table 18. The results of ranking options by the methods SAW, WASPAS, TOPSIS, VIKOR, MOORA, COPRAS, and PIV of the study No. [30] have also been summarized in this table.

Table 18. Results of ranking options according to different methods

No.	R	*CURLI*	*SAW [30]*	*WASPAS [30]*	*TOPSIS [30]*	*VIKOR [30]*	*MOORA [30]*	*COPRAS [30]*	*PIV [30]*
A_1	6	*8*	7	8	8	8	9	9	8
A_2	9	*9*	9	9	9	9	8	7	9
A_3	5	*5*	2	2	2	5	2	2	2
A_4	7	*6*	8	7	7	6	7	8	7
A_5	8	*7*	6	6	3	7	3	4	3
A_6	3	*3*	5	5	4	3	4	3	5
A_7	4	*4*	3	3	5	4	5	5	6
A_8	2	*2*	4	4	6	2	6	6	4
A_9	1	*1*	1	1	1	1	1	1	1

From Table 18, it can be seen that the ranking results of the alternatives are not the same when implemented by different methods. This has also been confirmed in many studies [21-26]. However, both the R and CURLI methods show that A9 is the best option. Especially, A9 was also confirmed to be the best option when using SAW, WASPAS, TOPSIS, VIKOR, MOORA, COPRAS, and PIV methods [30]. Furthermore, the worst alternative shown when using these two methods is option A2, which is similar to when using the SAW, WAPSAS, TOPSIS, VIKOR, and PIV methods. Thus, through this example, it is once again confirmed that the two methods R and CURLI are equally effective in multi-criteria decision making.

3.3 Example 3

Data on the criteria of the bridge construction options were used in this study [31]. With six options available, each option is evaluated through seven criteria including: Access roads (C1), Scope of work on site arrangement (C2), Properties of banks (C3), Width of water barrier (C4), Masking conditions (C5), Scope of works on joining access roads with the crossing point (C6), and Protection of units (C7). In which C1, C3, C5, C7 are the criteria as large as possible. In contrast, C2, C4, and C6 are criteria as small as possible. Of these seven criteria, only criterion C4 is represented by numbers (quantitative criteria), while the remaining five criteria are not represented by numbers (qualitative criteria), presented shown in Table 19. The abbreviations in this table are also explained at the end of this table. After surveying the opinions of experts, the priority of the criteria decreases in the following order C1 > C2 > C3 > C4 > C5 > C6 > C7 [31]. The ranking of the alternatives (Table 19) by the two methods R and CURLI is similar to that of example 1 and the results are presented in Table 20. The results of ranking options by MABAC method of study number [31] have also been summarized in this table.

Table 19. Data for example 3 [31]

No.	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A_1	M	L	E	50.33	M	VS	VB
A_2	G	S	M	43.33	VB	VL	G
A_3	VB	VL	G	51.33	E	S	M
A_4	B	M	VB	48.33	G	VS	VB
A_5	M	L	B	41.67	E	L	G
A_6	E	M	G	47.67	G	S	B
VB = Very Bad; B = Bad; M = Medium; G = Good; E = Excellent; VS = Very Small; S = Small; L = Large; VL = Very Large							

Table 20. Results of ranking options according to different methods

No.	R	*CURLI*	*MABAC [31]*
A_1	4	5	4
A_2	2	2	2
A_3	6	6	6
A_4	5	4	5
A_5	3	3	3
A_6	1	1	1

From Table 20, it can be seen that the rankings of 1, 2, 3 and 6 are exactly the same when ranked by the R method and the CURLI method. Moreover, the ranking results of these four alternatives are also completely coincidental when using the MABAC method. In addition, the ranking results of all six alternatives are completely identical for the R method and the MABAC method. From this, it can be concluded that the two methods R and CURLI are equally effective when ranking alternatives, and they are also equivalent to the MABAC method.

The results of comparing two methods R and CURLI in three examples with different fields, different number of options, different number of criteria and also different form of criteria (qualitative or quantitative form) for both R and CURLI method always determine the best solution. This result is also consistent with other methods. From this, a firm conclusion can be drawn that the R method and the CURLI method are equally effective in multi-criteria decision making. Both methods do not need to normalize the data and do not need to use additional weight determination methods. These characteristics make these two methods applicable to multi-criteria decision making for alternatives when the criteria of the alternatives are both qualitative and quantitative. When using the R method, it is necessary to consider the priority between the criteria. The priority of the criteria is completely dependent on the opinion of the decision maker, so when necessary, the opinion of experts should be taken into account. In contrast, the CURLI method is completely independent of the decision maker's point of view, but the ranking of alternatives is completely dependent on the criteria in each option. The aforementioned characteristics of these two methods provide recommendations for choosing one of them. In particular, when the decision maker wants to impose preference on one criterion over another, the R method should be used. In contrast, the CURLI method should be used.

4. CONCLUSION

The choice of a multi-criteria decision-making method, a data normalization method, and a weighting method is complex. An incorrect choice will lead to erroneous results. If multi-criteria decision making does not include the above tasks, it will greatly reduce the cost of decision making as well as increase the reliability of the work. Two methods R and CURLI are multi-criteria decision making methods that satisfy that problem. The first contribution of this study is to clarify the steps when applying the R method by building calculation formulas instead of paragraphs as suggested by this method. The effectiveness of these two methods has been compared in three specific cases in three different domains. These two methods have similar accuracy and are also equivalent to other methods (CODAS, SAW, WASPAS, TOPSIS, VIKOR, MOORA, COPRAS, PIV, MABAC). This is the second novelty discovered in this study. When it is necessary to reduce the time to choose the method of data normalization or the method of determining the weight of the criteria, R and CURLI are two methods that can be used. These two methods have a prominent advantage when the criteria are qualitative. In cases where it is desirable to impose a preference between criteria, the R method should be used. On the contrary, the CURLI method should be used when priority among criteria is not concerned. The hardest part of this study was probably the manual method used to sort the rows and columns in the process scoring matrix to find the best solution (in the CURLI method). That will be more complicated when the number of options is large. Computer programming to sort the rows and columns eliminates this hassle. This is the work that will be done in the near future.

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