NUMERICAL MODELING OF THE CRACK PROPAGATION PARAMETERS OF TWO DIFFERENT ELEMENTS BY THE FEM METHOD

UDC:539.42:519.6
Original scientific paper https://doi.org/10.46793/adeletters.2024.3.1.5

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Abstract:
Fracture mechanics is fundamental in various fields, such as mechanical engineering, civil engineering, hydraulics, and medicine. In addition, thanks to this field, we can estimate the age of the components of a structure, and the inspection and maintenance intervals can be precise. Thus, fracture mechanics is a science that studies numerical tools to characterize various parameters, such as the contour integral (J), stress intensity factors, and internal energy. However, in this paper, comparing the two types of elements (CPS3) and (CPS4R) gives comparable and proportional results; logically, a good correlation was obtained between them. In this article, those parameters were simulated and analyzed numerically by the finite element method (FEM) of a two-dimensional model consisting of a steel material with elastic properties. The analysis of the crack parameters was evaluated by the two models of elements CPS3 and CPS4R. On the other hand, the crack parameters between the two elements were compared. In addition, the numerical simulation was carried out using the computer code ABAQUS 16.3.1.

ARTICLE HISTORY
Received: 9 January 2024
Revised: 26 February 2024
Accepted: 21 March 2024
Published: 31 March 2024

KEYWORDS
CPS3, CPS4R, finite element method (FEM), J-integral stress intensity factor (SIF), fracture mechanics

1. INTRODUCTION

Today, numerical simulation plays an essential role in modeling to solve problems in modern technology; this modeling is done by numerical methods, such as the finite element method, the extended finite element method (X-FEM), etc. In the research [1], the 2D extended finite element method (X-FEM) was used to model crack propagation and energy evaluation (ALLSE) by simulation ABAQUS software.

Regarding crack growth, Lee et al. [2] developed a new advanced method (AI-FEM). The AI-FEM method used the simulation program to calculate arbitrary structures' exact stress intensity factor. Alkaissi [3] used the Abaqus software FEM method to analyze the crack propagation.

Laribou and Qotni's [4] analytical calculations were examined and verified the SIF through an empirical approach of the form factor in mode *I*. Laribou and Qotni used the finite element method with the Abaqus software for the two different cracks in the linear elastic domain [4]. They applied the method to a crack with a circular cross-section and an elliptical crack under a uniform tensile load. Fakkoussi et al. [5] it was calculated the stress intensity factor K_I, in mode *I*, by the FEM and the X-FEM in the linear elastic domain of a longitudinal semi-elliptical crack of a tube. In the research [6], a formulation was proposed of the FEM method to analyze crack propagation problems. In addition, a combined comparative study between the two experimental and numerical techniques was presented in research [7] to study the fracture properties of additively fabricated polymer parts using digital image correlation measurement (DIC). In this field, Shafiei [8] studied the influence of geometry on the growth of an inclined crack. Moreover, he studied the path for an inclined crack subjected to dynamic loading for different

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angles and stress values. Bentahar and Benzaama [9] presented a numerical study to evaluate different crack parameters of a multi-position FEM model. Tu et al. [10] studied the crack growth behavior in a cortex structure subjected to low cycle disproportionate biaxial stress during different stages. The research [11] presents a complete finite element formulation of complete coupling of the thermomechanical problem of cracked bodies. Furthermore, based on the stretching finite element method (SFEM), another method makes it possible to evaluate the stress intensity factor SIF of an initial crack propagation problem presented in research [12,13].

2. FRACTURE MECHANICS

The 4-node bilinear plane stress quadrilateral (CPS4R) and the 3-node linear plane stress triangle of (CPS3) elements were used around the crack front area called the singularity zone for the 2D models, Fig. 1a). This type of element is well-suited for simulation. The types of these “quarter point” singular elements are reduced quadratic elements. The Fig. 1b) shows different contours around the crack tip.

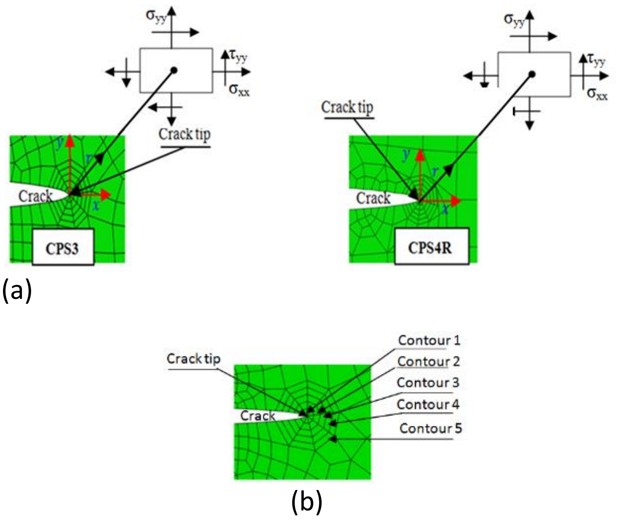

Fig. 1. a) Stress field around the crack front and b) Definition of the different contours

2.1 Law of Fatigue Crack Propagation

Paris and Erdogan [14] proposed the fatigue law to model the crack propagation in the two-dimensional case. This law is based on constant amplitude tests for which the propagation speeds appear as a linear function of the variation of the SIF in a log-log diagram, Fig. 2.

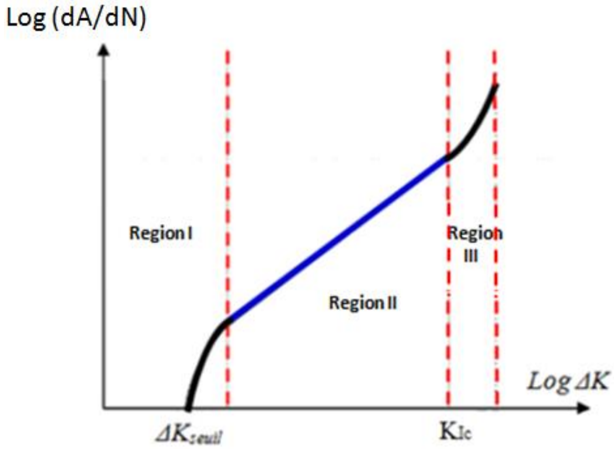

Fig.2. Schematic illustration of the three modes of propagation of a mode (*I*) crack

In region I, the crack growth rates are minimal.

The joining of microcracks and the formation of one or more macrocracks were observed; for this the value of *ΔK* is greater than (*ΔK*) threshold.

In region II, the crack rate is linear. In this regime on the log-log diagram, the Paris law, defined by equation (1), establishes a linear relationship between *Log* (*dA/dN*) and *Log ΔK*; this is the so-called stable propagation regime.

$$\frac{dA}{dN} = C\left(\Delta K\right)^m, \tag{1}$$

where: *ΔK* being the variation of the stress intensity factor during a cycle, which induces an advance (dA) of the crack, *C* and *m* are the two parameters of the material defining respectively the position and the slope of the Paris line.

2.2. Illustration of the Stress Distribution at the Crack Tip

The general equation of the stress field in 2D near the crack front, defined by the *K* stress intensity factor, is given by [15]:

$$\sigma_{i,j}^{I,II}(r,\theta) = \frac{K_{I,II}}{\sqrt{2\pi r}} f_{ij}(\theta) \tag{2}$$

where $K_{I,II}$ is SIF in modes I and II respectively, $\sigma_{i,j}^{I,II}$ is the associated stress field with these modes. It is also the stress field associated with the first and second modes. K_I is the factor that contributes significantly to determining the stress state at the crack end level, which tends to propagate following the direction orthogonal to the maximum tangential stress at its end [16].

$$\sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$
$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \quad (3)$$
$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}$$

2.3 Stress Intensity Factor

The stress intensity factor is defined according to the following equation [17].

$$K_I = F\sigma\sqrt{a\pi} \quad (4)$$

$$F = 1.12 - 0.23(a/C) + 10.6(a/C)^2 - 21.7(a/C)^3 + 30.4(a/C)^4, \quad (5)$$

where, *F* is the geometric correction factor of the used model, *σ* is the applied stress, *a* is the crack length, and *C* is the length of the plate.

The stress intensity factor K_{II} is calculated by the equation:

$$K_I \sin\theta + K_{II}(3\cos\theta - 1) = 0 \quad (6)$$

where θ is the kinking angle during crack propagation.

2.4 Contour Integral (J)

Several authors in continuum mechanics have made it possible to model the problem of the presence of a crack in-depth and have developed calculation methods. These authors [18,19], with contour integrals *J*, [20,21] among others, introduced an arbitrary field in formulating the integral they approached. Indeed, the work was developed based on elasticity in small displacements and addressed the first phase of the cracking process.

3. NUMERICAL MODEL

In the researched working model, it was assumed that the plate is rectangular, with dimensions (*RxC*) length and width (16mm x 7mm) see Fig. 3a). The plate is made of A36 steel with a Young's modulus E=2·10^{11} Pa and a Poisson's ratio ν=0.3. The finite element code ABAQUS was used to calculate the various internal energies, stress intensity factors, and *J*-integral.

Uniform tensile stress *σ*=100 *MPa* was applied to the upper surface of the plate, and constraints were applied to the lower surface, Fig. 3b). Additionally, Fig. 4 illustrates the different elements used in this study and the different meshes: (Fig. 4a) the model by the CPS3 elements and (Fig. 4b) by the CPS4R elements.

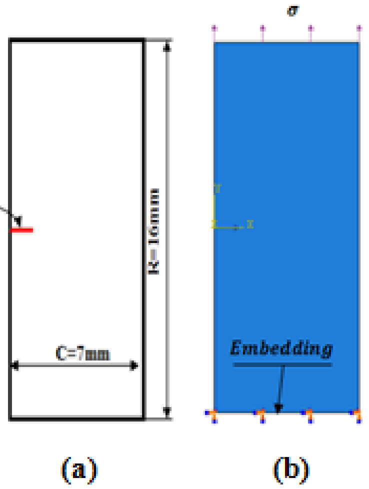

Fig. 3. Geometric characteristics of the model: a) Boundary conditions CPS3 elements and b) CPS4R elements

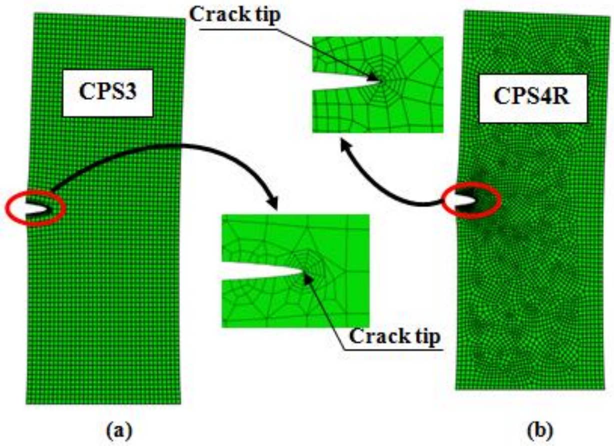

Fig. 4. Mesh FEM model: a) CPS3 elements and b) CPS4R elements

Fig. 5 shows the stress state at the singularity, the model by the CPS3 elements (Fig. 5a), and the model by the CPS4R elements (Fig. 5b).

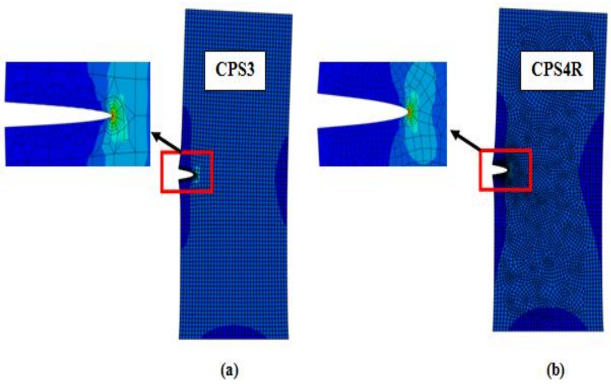

Fig. 5. Presentation of the state of the crack front of the FEM model: a) CPS3 elements and b) CPS4R elements

4. RESULTS AND DISCUSSIONS

Fig. 6 illustrates the evolution of the internal energy for the two models: the first mesh model, which consists of CPS3 elements, and the other model with CPS4R elements. Research [22] carried out the study of different energies at the crack front in the case analysis of the dissipation energy and [23,24] in the case of the evolution of the strain energy.

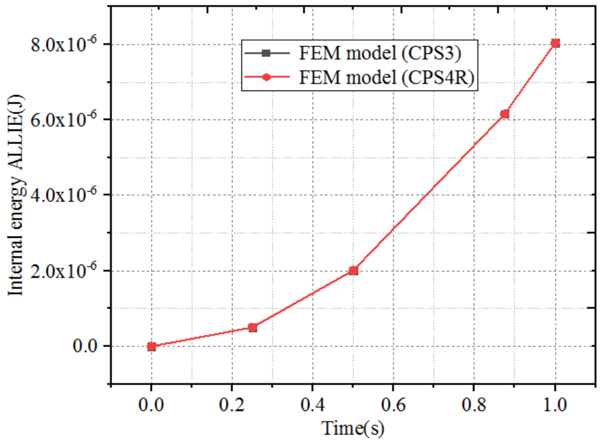

Fig. 6. Evolution of the internal energy (ALLIE) concerning two models

Research has shown that the comparison between the two models is proportional, and the internal energy is distributed uniformly along the initial crack. Based on the research, it can be observed that the energy evolution increases with increasing time, in the interval from 0 s to 1.0 *s*, and the interval of internal energy varies between 0 *J* and $8.0 \cdot 10^{-6}$ *J*.

Fig. 7 presents the evolution of the contour integral (*J*) as a function of time for the two models that we studied; we can see that the contour integral (*J*) increases from 0.25 s to 1.0 *s*.

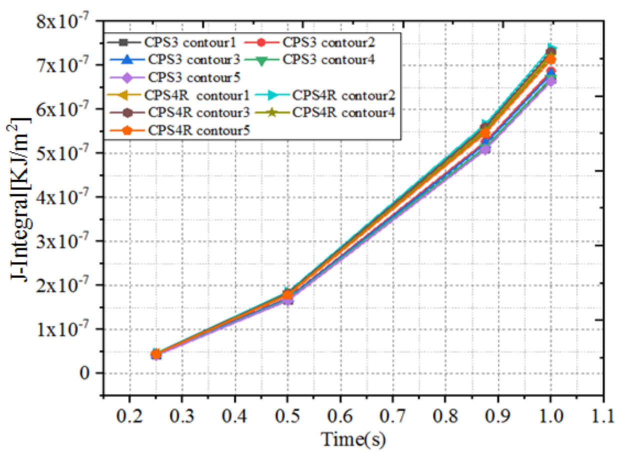

Fig. 7. Evolution of the contour integral (*J*) concerning the two models of the elements (CPS3) and (CPS4R)

Concerning the two types of elements, the increase in time causes an increase in the contour integral (*J*) concerning the two models, and the results obtained are proportional between the two types of elements.

Fig. 8 shows the results of the stress intensity factor K_I as a function of time by the FEM method.

In the time period from 0.35 s to 1.0 s, the CPS4R model has slightly higher values than the CPS3 model.

Furthermore, the stress intensity factor K_I is the same for both models in the period from 0.25 *s* to 0.35 *s*.

However, comparing the two types of elements (CPS3) and (CPS4R) yields comparable and proportional results; logically, a good correlation was obtained between them.

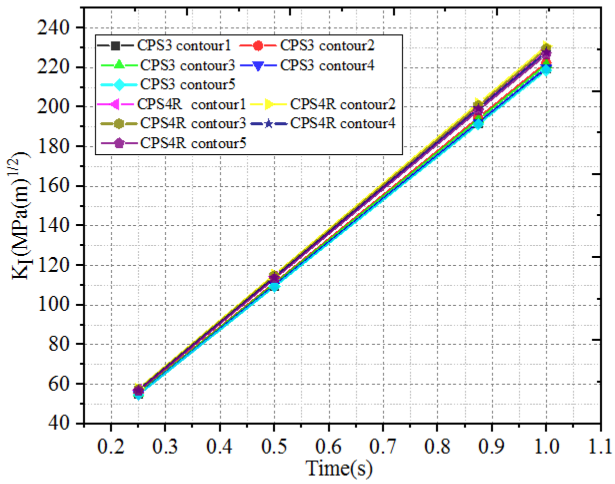

Fig. 8. Evolution of the stress intensity factor K_I concerning the two models of the elements (CPS3) and (CPS4R)

Fig. 9 shows the evolution of the stress intensity factor K_{II} as a function of time; from Fig. 9, it can be concluded that the results in the model containing elements (CPS3) decreased, and the values of K_{II} are confined between -1 MPa $(m)^{1/2}$ and -6.31 MPa $(m)^{1/2}$ in the interval of 0.25 *s* and 1.0 *s*. On the other hand, the results of K_{II} obtained by the CPS4R elements are increasing in the interval of the stress intensity factor vary between 0.2 MPa $(m)^{1/2}$ and 3 MPa $(m)^{1/2}$, with a time variation confined to the interval of 0.25 *s* and 1.0 *s*.

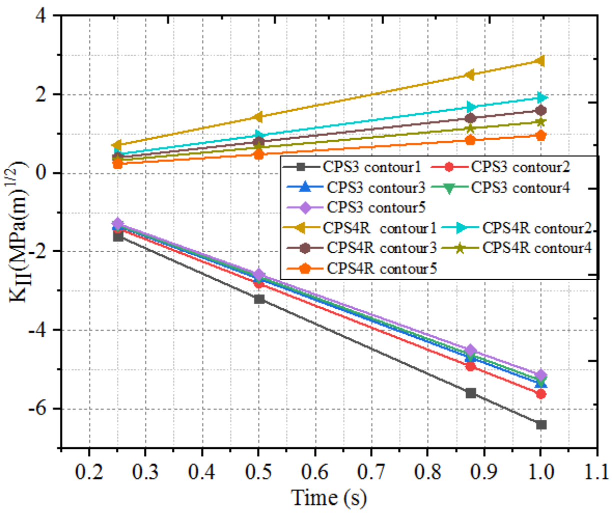

Fig. 9. Evolution of the stress intensity factor K_{II} concerning the two models of the elements (CPS3) and (CPS4R)

5. CONCLUSION

In this paper, two cases of modeling an initial crack were studied, on the one hand the modeling of a model which constitutes by the elements (CPS3) and the other study based on the study of a model which constitutes by the elements (CPS4R).

The finite element method was used for numerical modeling in both cases.

The comparison between the types of elements (CPS4) and (CPS4R) gives comparable and proportional results, and a good correlation was obtained between them.

It is observed that with the increase in time (*t*), the contour integral (*J*), the internal energy, the stress intensity factor K_I and the stress intensity factor K_{II} in the case of the model, which contained elements of (CPS4R) is increased.

The research showed that the stress intensity factor KII in the case of the model that contained elements of (CPS3) is reduced.

Indeed, the results concerning the study of crack propagation are always increased, except the results of K_{II} are always reduced.

In this study, the time interval ranged between 0.25 *s* and 1.0 *s*.

Conflicts of Interest

The author declares no conflict of interest.

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